

Possible physical self-asserting of the homogeneous  
vector potential  
A testing puzzle based on a G.P. Thomson-like arrangement

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**Abstract**

It is suggested a testing puzzle able to reveal the self-asserting property of the homogeneous vector potential field. As pieces of the puzzle are taken three reliable entities : (i) influence of a potential vector on the de Broglie wavelength (ii) a G.P. Thomson-like experimental arrangement and (iii) a special coil designed to create a homogeneous vector potential. The alluded property is not connected with magnetic fluxes surrounded by the vector potential field lines, but it depends on the fluxes which are outside of the respective lines. Also the same property shows that in the tested case the vector potential field is uniquely defined physical quantity, free of any adjusting gauge. So the phenomenology of the suggested quantum test differs on that of macroscopic theory where the vector potential is not uniquely defined and allows a gauge adjustment. Of course that the proposed test has to be subjected to adequate experimental validation.

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# 1 Introduction

The physical self-asserting (objectification) of the vector potential  $\vec{A}$  field, distinctly of electric and/or magnetic local actions, is known as Aharonov-Bohm Effect (ABE). It aroused scientific discussions for more than half a century ( see [1–8] and references). As a rule in ABE context the vector potential is curl-free field, but it is non-homogeneous (**n-h**) i.e. spatially non-uniform. In the same context the alluded self-asserting is connected quantitatively with magnetic fluxes surrounded by the lines of  $\vec{A}$  field. In the present paper we try to suggest a testing puzzle intended to reveal the possible physical self-asserting property of a homogeneous (**h**)  $\vec{A}$  field,. Note that in both **n-h** and **h** cases here we consider only fields which are constant in time.

The announced puzzle has as constitutive pieces three reliable Entities(**E**) namely :

- **E**<sub>1</sub>: The fact that a potential vector change the values de Broglie wavelength  $\lambda^{dB}$  of electrons. ■
- **E**<sub>2</sub>: An experimental arrangement of G. P. Thomson type, able to monitor the mentioned  $\lambda^{dB}$  values.■
- **E**<sub>3</sub>: A feasible special coil designed so as to create a **h**- $\vec{A}$  field. ■

Accordingly, in its wholeness, the puzzle has to put together the mentioned entities and, consequently, to synthesize a clear verdict regarding the alluded property of a **h**- $\vec{A}$  field .

Experimental setup of the suggested puzzle is detailed in the next Section 2. Essential theoretical considerations concerning the action of a **h**- $\vec{A}$  field are given in Section 3. The above noted considerations are fortified in Section 4 through a set of numerical estimations for the quantities aimed to be measured by means of the puzzle. Some concluding thoughts regarding a possible positive result of the suggested puzzle close the principal body of the paper in Section 5. Constructive and computational details regarding the special coil designed to generate a **h**- $\vec{A}$  field are presented in the Appendix.

## 2 Setup details of the experimental puzzle

The setup of the suggested experimental puzzle is pictured and detailed below in Fig. 1. It consists in a G. P. Thomson-like arrangement partially located in an area with a **h**- $\vec{A}$  field . The alluded arrangement is inspired from some illustrative figures [9, 10] about G. P. Thomson's original experiment and it disposes in a straight line the following elements: electrons source,

electrons beam , crystalline grating and detecting screen. An area with a  $\mathbf{h}-\vec{A}$  field can be obtained through a certain special coil whose constructive and computational details are given in the alluded Appendix.

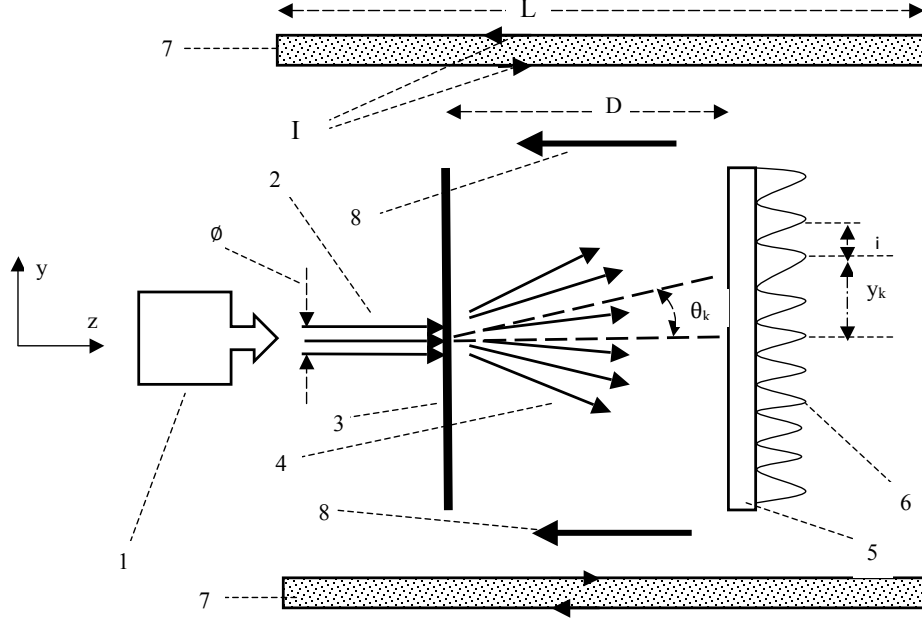


Figure 1: **Plane section in the image of suggested experimental puzzle setup, accompanied by the following explanatory records**  
1 – Source for a beam of mono-energetic and parallel moving electrons; 2 – Beam of electrons in parallel movements; 3 - Thin crystalline foil as diffraction grating; 4 – Diffracted electrons; 5 – Detecting screen; 6 Fringes in the plane section of diffraction pattern ; 7 – Special coil able to create a  $\mathbf{h} - \vec{A}$  field; 8 –  $\mathbf{h} - \vec{A}$  field ;  $\phi$  = the width of the electrons beam with  $\phi \gg a$  ( $a$  = interatomic spacing in crystal lattice of the foil -3);  $\theta_k$  = diffraction angle for the  $k$ -th order fringe ( $k = 0, 1, 2, 3, \dots$ ) ;  $y_k$  = displacement from the center line of the  $k$ -th order fringe ;  $i$  = interfringe width =  $y_{k+1} - y_k$ ;  $D$  = distance between crystalline foil and screen ( $D \gg \phi$ );  $L$  = length of the special coil ( $L \gg D$ ) ;  $I$  = intensity of current in wires of the coil.

The explanatory records accompanying Fig. 1 have to be supplemented with the next notes:

• **Note 1** : If in Fig. 1 are omitted the elements 7 and 8 ( i.e. the sections in special coil and the lines of  $\mathbf{h}-\vec{A}$  field ) one obtains a G. P. Thomson-like arrangement as it is illustrated in references [9, 10]. ■

• **Note 2** : Evidently the above mentioned G. P. Thomson-like arrangement is so designed and constructed that it can be placed inside of a vacuum

glass container. The respective container is not showed in Fig. 1 and it will leave out the special coil. ■

• **Note 3** : At the incidence on crystalline foil the electrons beam must ensure a coherent and plane front of de Broglie waves. Similar ensuring is required [11] for optical diffracting waves at the incidence on a classical diffraction grating. ■

• **Note 4** : In Fig. 1 the detail 6 displays only the linear projections of the fringes from the diffraction pattern. In its wholeness the respective pattern consists in a set of concentric circular fringes (diffraction rings). ■

### 3 Theoretical considerations concerning action of a $\mathbf{h}\text{-}\vec{A}$ field

The leading idea of the above suggested puzzle is to search the possible changes caused by a  $\mathbf{h}\text{-}\vec{A}$  field in diffraction of quantum (de Broglie) electronic waves. That is why now firstly we remind some quantitative characteristics of the diffraction phenomenon.

The most known scientific domain where the respective phenomenon is studied regards the optical light waves [11]. In the respective domain one uses as main element the so called "*diffraction grating*" i.e. a piece with periodic structure having slits separated by distances  $a$  and which diffracts the light into beams in different directions. For a light normally incident on such an element the grating equation (condition for intensity maximum) has the form :  $a \cdot \sin \theta_k = k\lambda$ , where  $k = 0, 1, 2, \dots$ . In the respective equation  $\lambda$  denotes the light wavelength and  $\theta_k$  is the angle at which the diffracted light have the  $k - th$  order maximum. If the diffraction pattern is received on a detecting screen the  $k - th$  order maximum appear on the screen in position  $y_k$  given by relation  $\tan \theta_k = (y_k/D)$ , where  $D$  denote the distance between screen and grating. For the distant screen assumption, when  $D \gg y_k$ , can be written the relations:  $\sin \theta_k \approx \tan \theta_k \approx (y_k/D)$ . Then, with regard to the mentioned assumption, one obtains that diffraction pattern on the screen is characterized by an interfringe distance  $i = y_{k+1} - y_k$  given through the relation

$$i = \lambda \frac{D}{a} \quad (1)$$

Note the fact that the above quantitative aspects of diffraction have a generic character, i.e. they are valid for all kinds of waves including the de Broglie ones. The respective fact is presumed as a main element of the experimental puzzle suggested in the previous section. Another main element of the alluded puzzle is the largely agreed idea [1–8] that the de Broglie

electronic wavelength  $\lambda^{dB}$  is influenced by the presence of a  $\vec{A}$  field. Based on the two before mentioned main elements the considered puzzle can be detailed as follows.

In experimental setup depicted in Fig. 1 the crystalline foil 3 having interatomic spacing  $a$  plays the role of a diffraction grating. In the same experiment on the detecting screen 5 is expected to appear a diffraction pattern of the electrons. The respective pattern would be characterized by an interfringe distance  $i^{dB}$  definable through the formula  $i^{dB} = \lambda^{dB} \cdot (D/a)$ . In that formula  $D$  denote distance between crystalline foil and screen, supposed to satisfy the condition  $D \gg \phi$ , where  $\phi$  represents the width of the incident electrons beam. In absence of a  $\mathbf{h}\text{-}\vec{A}$  field the  $\lambda^{dB}$  of a non-relativistic electron is known as having the expressions:

$$\lambda^{dB} = \frac{h}{p_{mec}} = \frac{h}{mv} = \frac{h}{\sqrt{2m\mathcal{E}}} \quad (2)$$

In the above expressions  $h$  is the Planck's constant while  $p_{mec}$ ,  $m$ ,  $v$  and  $\mathcal{E}$  denote respectively the mechanical momentum, mass, velocity and kinetic energy of the electron. If the alluded energy is obtained in the source of electrons beam (i.e. piece 1 in Fig. 1) under the influence of an accelerating voltage  $U$  one can write  $\mathcal{E} = e \cdot U$  and  $p_{mec} = mv = \sqrt{2meU}$ .

Now, in connection with the situation depicted in Fig. 1, let us look for the expressions of the electrons characteristic  $\lambda^{dB}$  and respectively of  $i^{dB} = \lambda^{dB} \cdot (D/a)$  in presence of a  $\mathbf{h}\text{-}\vec{A}$  field. Firstly we note the known fact [6] that a particle with the electrical charge  $q$  and the mechanical momentum  $\vec{p}_{mec} = m\vec{v}$  in a potential vector  $\vec{A}$  field acquires an additional (*add*) momentum,  $\vec{p}_{add} = q\vec{A}$ , so that its "effective" (*eff*) momentum is  $\vec{P}_{eff} = \vec{p}_{mec} + \vec{p}_{add} = m\vec{v} + q\vec{A}$ . Then for the electrons ( with  $q = -e$ ) supposed to be implied in the experiment depicted in Fig. 1 one obtains the effective (*eff*) quantities

$$\lambda_{eff}^{dB}(A) = \frac{h}{mv + eA} ; \quad i_{eff}^{dB}(A) = \frac{hD}{a(mv + eA)} \quad (3)$$

Further on we have to take into account the fact that the  $\mathbf{h}\text{-}\vec{A}$  field acting in the discussed experiment is generated by a special coil whose plane section is depicted by the elements 7 from Fig. 1. Then from the relation (10) established in Appendix we have  $A = \mathcal{K} \cdot I$ , where  $\mathcal{K} = \frac{\mu_0 N}{2\pi} \cdot \ln\left(\frac{R_2}{R_1}\right)$ . Add here the fact that in the considered experiment  $mv = \sqrt{2meU}$ . Then for the effective interfringe distance  $i_{eff}^{dB}$  of diffracted electrons one find

$$i_{eff}^{dB}(A) = i_{eff}^{dB}(U, I) = \frac{hD}{a\left(\sqrt{2meU} + e\mathcal{K}I\right)} \quad (4)$$

respectively

$$\frac{1}{i_{eff}^{dB}(U, I)} = f(U, I) = \frac{a\sqrt{2me}}{hD}\sqrt{U} + \frac{ae\mathcal{K}}{hD}I \quad (5)$$

## 4 A set of numerical estimations

The verisimilitude of the above suggested testing puzzle can be fortified to some extent by transposing several of the previous formulas in their corresponding numerical values. For such a transposing firstly we will appeal to numerical values known from G.P. Thomson-like experiments. So, as regards the elements from Fig. 1 we quote the values  $a = 2.55 \cdot 10^{-10}m$  (for a crystalline foil of copper) and  $D = 0.1m$ . As regards  $U$  we take the often quoted value:  $U = 30 \cdot kV$ . Then the mechanical momentum of the electrons will be  $p_{mec} = mv = \sqrt{2meU} = 9.351 \cdot 10^{-23}kg \cdot m \cdot s^{-1}$ . The additional (add) momentum of the electron, induced by the special coil, is of the form  $p_{add} = e\mathcal{K} \cdot I$  where  $\mathcal{K} = \frac{\mu_0 N}{2\pi} \cdot \ln\left(\frac{R_2}{R_1}\right)$ . In order to estimate the value of  $\mathcal{K}$  we propose the following practically workable values:  $R_1 = 0.1m$ ,  $R_2 = 0.12m$ ,  $N = 2\pi R_1 \cdot n$  with  $n = 2 \cdot 10^3 m^{-1}$  = number of wires (of  $1mm$  in diameter) per unit length, arranged in two layers. With the well known values for  $e$  and  $\mu_0$  one obtains  $p_{add} = 7.331 \cdot 10^{-24}(kg \cdot m \cdot s^{-1} \cdot A^{-1}) \cdot I$  (with  $A = ampere$ ).

For wires of  $1mm$  in diameter, by changing the polarity of voltage powering the coil, the current  $I$  can be adjusted in the range  $I \in (-10 \text{ to } +10)A$ . Then the effective momentum  $\vec{P}_{eff} = \vec{p}_{mec} + \vec{p}_{add}$  of the electrons have the values within the interval  $(2.040 \text{ to } 16.662) \cdot 10^{-23}kg \cdot m \cdot s^{-1}$ . Consequently, due to the above mentioned values of  $a$  and  $D$ , the effective interfringe distance  $i_{eff}^{dB}$  defined in (4) changes in the range  $(1.558 \text{ to } 12.725)mm$ , respectively its inverse from (5) has values within the interval  $(78.58 \text{ to } 641.84)m^{-1}$ .

Now note that in absence of  $\mathbf{h}\vec{A}$  field (i.e. when  $I = 0$ ) the interfange distance  $i^{dB}$  specific to a simple G.P. Thomson experiment has the value  $i^{dB} = \frac{hD}{a\sqrt{2meU}} = 2.776mm$ . Such a value is within the values range of  $i_{eff}^{dB}$  characterizing the presence of a  $\mathbf{h}\vec{A}$  field. This means that the quantitative evaluation of the mutual relationship of  $i_{eff}^{dB}$  versus  $I$  and therefore of the self-asserting of a  $\mathbf{h}\vec{A}$  field can be done with techniques and accuracies similar to those for simple G.P. Thomson experiment.

## 5 Some concluding remarks

The aim of the experimental puzzle suggested above is to test a possible physical self-asserting for a  $\mathbf{h}\text{-}\vec{A}$  field. Such a test can be done concretely by comparative measurements of the interfringe distance  $i_{eff}^{dB}$  and of the current  $I$ . Additionally it must to examine whether the results of the mentioned measurements verify the relations (4) and (5) ( particularly according to (5) the quantity  $(i_{eff}^{dB})^{-1}$  is expected to show a linear dependence of  $I$ ). If the above outcomes are positive one can be notified the fact that a  $\mathbf{h}\text{-}\vec{A}$  field has its own characteristic of physical self-asserting. Such a fact leads in one way or another to the following remarks (**R**):

- **R<sub>1</sub>**: The self-asserting of  $\mathbf{h}\text{-}\vec{A}$  field differs from the one of  $\mathbf{n}\text{-}\mathbf{h}\text{-}\vec{A}$  field which appears in ABE. This because, by comparison with the illustrations from [12], one can see that : (i) by changing of  $\mathbf{n}\text{-}\mathbf{h}\text{-}\vec{A}$  the diffraction pattern undergoes a simple translation on the screen, without any modification of interfringe distance, while (ii) according to the relations (4) and (5) a change of  $\mathbf{h}\text{-}\vec{A}$  (by means of current  $I$ ) does not translate the diffraction pattern but varies the associated interfringe distance. The mentioned variation is similar with those induced [12] by changing (through accelerating voltage  $U$ ) the values of mechanical momentum  $\vec{p}_{mec} = m\vec{v}$  for electrons. ■

- **R<sub>2</sub>** : There are a difference between the objectification (self-asserting) of  $\mathbf{h}\text{-}\vec{A}$  and  $\mathbf{n}\text{-}\mathbf{h}\text{-}\vec{A}$  fields in relation with the magnetic fluxes surrounded or not by the field lines. The difference is pointed out by the next aspects:

- (i) On the one hand, as it is known from ABE, in case of a  $\mathbf{n}\text{-}\mathbf{h}\text{-}\vec{A}$  field the self-asserting is in a direct dependence on magnetic fluxes surrounded by the field lines.

- (ii) On the other hand the self-asserting of a  $\mathbf{h}\text{-}\vec{A}$  field is not connected with magnetic fluxes surrounded by the field lines. But note that due to the relations (4) and (5) the respective self-asserting appears to be dependent (through the current  $I$ ) on magnetic fluxes not surrounded by field lines of the  $\mathbf{h}\text{-}\vec{A}$ . ■

- **R<sub>3</sub>** : Another particular characteristic of the self-asserting forecasting above for  $\mathbf{h}\text{-}\vec{A}$  is that in the proposed test the vector potential field appears as an uniquely defined physical quantity free of any adjusting gauge. So the phenomenology of the suggested test differs on that of macroscopic situations where [13,14] the vector potential is not uniquely defined and allows a gauge adjustment. Surely that such a fact (difference) and its implications have to be approached in more elaborated studies.

## Postscript

As presented above the suggested puzzle and its positive result appear as purely hypothetical things, despite of the fact that they are based on the essentially reliable entities (constitutive pieces) presented in Introduction. Of course that a true confirmation of the alluded result can be done by an action of putting in practice the whole puzzle. Unfortunately I do not have access to material logistics able to allow me an effective practical test of the puzzle in question. That is why I warmly appeal to experimentalist researchers that have adequate logistics to put in practice the suggested test and to verify its validity.

## Appendix

### Constructive and computational details for a special coil able to create a $\mathbf{h}\text{-}\vec{A}$ field

#### The case of an ideal coil

An experimental area of macroscopic size with a  $\mathbf{h}\text{-}\vec{A}$  field can be realized with the aid of a special coil whose constructive and computational details are presented below. The announced details are improvements of ideas promoted by us in an early preprint [15].

The basic element in designing of the mentioned coil is the  $\mathbf{h}\text{-}\vec{A}$  field generated by a rectilinear infinite conductor carrying a direct current. If the conductor is located along the axis Oz and current have the intensity I, the Cartesian components ( written in SI units) of the mentioned  $\mathbf{h}\text{-}\vec{A}$  field are given [16] by formulas:

$$A_x(1) = 0 \quad A_y(1) = 0 \quad A_z(1) = -\mu_0 \frac{I}{2\pi} \ln r \quad (6)$$

Here  $r$  denote the distance from the conductor of the point where  $\mathbf{h}\text{-}\vec{A}$  is evaluated and  $\mu_0$  is vacuum permeability.

Note that formulas (6) are of ideal essence because they describe a  $\mathbf{h}\text{-}\vec{A}$  field generated by an infinite (ideal) rectilinear conductor. Further firstly we will use the respective formulas in order to obtain the  $\mathbf{h}\text{-}\vec{A}$  field generated by an ideal annular coil. Later one we will specify the conditions in which the results obtained for the ideal coil can be used with good approximation in the characterization of a real ( non-ideal) coil of practical interest for the puzzle-experiment suggested and detailed in Sections 2,3 and 4.

The mentioned special coil has the shape depicted in Fig. 2-(a) (i.e. it is a toroidal coil of rectangular section). In the respective figure the finite



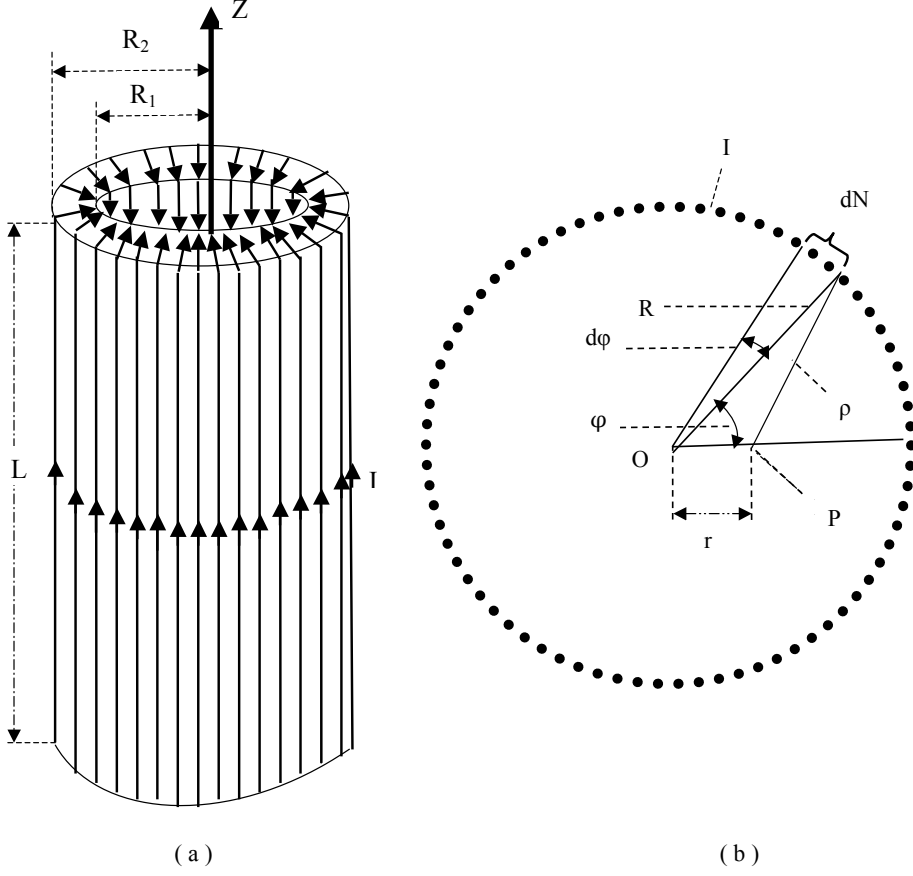


Figure 2: Schemes for an annular special coil

quantities  $R_1$  and  $R_2$  represent the inside and outside finite radii of coil while  $L \rightarrow \infty$  is the length of the coil. For evaluation of the  $\mathbf{h}\text{-}\vec{A}$  generated inside of the mentioned coil let us now consider an array of infinite rectilinear conductors carrying direct currents of the same intensity  $I$ . The conductors are mutually parallel and uniformly disposed on the circular cylindrical surface with the radius  $R$ . Also the conductors are parallel with  $Oz$  as symmetry axis. In a cross section the considered array are disposed on a circle of radius  $R$  as can be seen in Fig. 2-(b). On the respective circle the azimuthal angle  $\varphi$  locate the infinitesimal element of arc whose length is  $Rd\varphi$ . On the respective arc is placed a set of conductors whose number is  $dN = \left(\frac{N}{2\pi}\right) d\varphi$ , where  $N$  represents the total number of conductors in the whole considered array. Let be an observation point  $P$  situated at distances  $r$  and  $\rho$  from the center  $O$  of the circle respectively from the infinitesimal arc (see the Fig. 2-(b)). Then, by taking into account (6), the  $z$ -component of the  $\mathbf{h}\text{-}\vec{A}$  field generated

in  $P$  by the  $dN$  conductors is given by relation

$$A_z(dN) = A_z(1) dN = -\mu_0 \frac{NI}{4\pi^2} \ln \rho \cdot d\varphi \quad (7)$$

where  $\rho = \sqrt{(R^2 + r^2 - 2Rr \cos \varphi)}$ . Then the all  $N$  conductors will generate in the point  $P$  a  $\mathbf{h}\text{-}\vec{A}$  field whose value  $A$  is

$$A = A_z(N) = -\mu_0 \frac{NI}{8\pi^2} \int_0^{2\pi} \ln(R^2 + r^2 - 2Rr \cos \varphi) \cdot d\varphi \quad (8)$$

For calculating the above integral can be used formula (4.224-14) from [17]. So one obtains

$$A = -\mu_0 \frac{NI}{2\pi} \ln R \quad (9)$$

This relation shows that the value of  $A$  does not depend on  $r$ , that is on the position of  $P$  inside the circle of radius  $R$ . Accordingly this means that inside the respective circle the potential vector is homogeneous. Then starting from (9), one obtains that the inside space of an ideal annular coil depicted in Fig. 2-(a) is characterized by a  $\mathbf{h}\text{-}\vec{A}$  field whose value is

$$A = \mu_0 \frac{NI}{2\pi} \ln \left( \frac{R_2}{R_1} \right) \quad (10)$$

### From the ideal coil to a real one

The above presented coil is of ideal essence because their characteristics were evaluated on the base of ideal formulas (6). But in practical matters, such is the puzzle-experiment proposed in Sections 2 and 3, one needs of a real coil which may be effectively constructed in a laboratory. That is why it is important to specify the main conditions in which the above ideal results can be used in real situations. The mentioned conditions are displayed here below.

- *On the geometrical sizes* : In laboratory it is not possible to operate with objects of infinite sizes. Then it must to note the restrictive conditions so that the characteristics of the ideal coil discussed above to remain as good approximations for a real coil of similar geometric form. In the case of a finite coil having the form depicted in the Fig. 2-(a) the alluded restrictive conditions impose the relations  $L \gg R_1$ ,  $L \gg R_2$  and  $L \gg (R_2 - R_1)$ . If the respective coil is regarded as a piece in the puzzle-experiment from Fig. 1 there are indispensable the relations  $L \gg D$  and  $L \gg \phi$ .

- *About the marginal fragments* : In principle the marginal fragments of coil (of widths  $(R_2 - R_1)$ ) can have disturbing effects on the Cartesian components of  $\vec{A}$  inside the the space of practical interest. Note that, on the one hand, in the above mentioned conditions  $L \gg R_1$ ,  $L \gg R_2$  and  $L \gg (R_2 - R_1)$  the alluded effects can be neglected in general practical affairs. On the other hand in the particular case of the proposed coil the alluded effects are also diminished by the symmetrical flowings of currents in the respective marginal fragments.

- *As concerns the helicity* : The discussed annular coil is supposed to be realized by turning a single piece of wire. The spirals of the respective wire are not strictly parallel with the symmetry axis of the coil (  $Oz$  axis) but they have a certain helicity (corkscrew-like path). Of course that the alluded helicity has disturbing effects on the components of  $\vec{A}$  inside the coils. Note that the mentioned helicity-effects can be diminished (and practically eliminated) by using an idea noted in another context in [18]. The respective idea proposes to arrange the spirals of the coil in an even number of layers, the spirals from adjacent layers having equal helicity but of opposite sense.

## References

- [1] Y. Aharonov and D. Bohm, Significance of electromagnetic potentials in the quantum theory, *Phys.Rev.***115** (1959) 485-491.
- [2] Y. Aharonov, D. Bohm, Further Considerations on Electromagnetic Potentials in the Quantum Theory, *Phys.Rev.* **123** (1961)1511-1524.
- [3] . S.Olariu, I. I.Popescu, The quantum effects of electromagnetic fluxes, *Rev. Mod. Phys.* **57** (1985) 339-436.
- [4] M. Peshkin, A. Tonomura, The Aharonov-Bohm Effect, *Lecture Notes in Physics* (Springer) **340** (1989) 1-152.
- [5] M. Dennis, S. Popescu, L. Vaidman, Quantum Phases: 50 years of the Aharonov-Bohm effect and 25 years of the Berry phase, *J. Phys. A: Math. Theor.* **43** (2010) 350301
- [6] A. Ershkovich, Electromagnetic potentials and Aharonov-Bohm effect, arXiv:1209.1078v2, last revised 10 Apr 2013
- [7] V. A. Leus, R.T. Smith, S. Maher, The Physical Entity of Vector Potential in Electromagnetism, *Appl. Phys.Research*, **5** ( 2013) 56 - 68.

- [8] B. J. Hiley, The Early History of the Aharonov-Bohm Effect, arXiv:1304.4736v1.
- [9] Images for G.P. Thomson experiment:  
`<https://www.google.ro/search?q=g+p+thomson+experiment>`  
`<&tbm=isch&tbo=u&source=univ&sa=X&ei=SHnnUtj2CsiAyAPk1oDwAQ>`  
`<&ved=0CDAQsAQ&biw=915&bih=737>` , accesed 1/28/2014.
- [10] T. A. Arias, G.P. Thomson Experiment  
`<\protect\vrule width0pt\protect\href{http://muchomas.lassp.cornell.edu/}{ht`  
`<8.04/1997/quiz1/node4.html>` , accesed 1/28/2014.
- [11] M. Born , E. Wolf, *Principle of Optics, Electromagnetic theory of propagation, interference and diffraction of light*, Seventh (Expanded) Edition (Cambridge University Press, Reprinted 2003)
- [12] S.M. Blinder, Aharonov- Bohm Effect, From the Wolfram Demonstrations Project <http://www.youtube.com/watch?v=OgDPK5MLVnE>, accesed 2/28/2014
- [13] J.D. Jackson , *Classical Electrodynamics* (John Wiley, N.Y. 1962)
- [14] L. Landau, E. Lifchitz, *Theorie des Champs* (Ed. Mir, Moscou 1970)
- [15] S. Dumitru, M. Dumitru, Are there observable effects of the vector potential? : a suggestion for probative experiments of a new type (different from the proposed Aharonov-Bohm one) CERN Central Library PRE 24618 , Barcode 38490000001, Jan 1981. - 20 p.
- [16] R.P. Feynman, R.B. Leighton, M. Sands, *The Feynman Lectures on Physics, vol. II* (Addison-Wesley, Reading Mass. 1964).
- [17] I.S. Gradshteyn, I.M. Ryzhik, *Table of Integrals, Series, and Products, Seventh Edition* ( Elsevier,2007).
- [18] O. Costa de Beauregard, J. M. Vigoureux, Flux quantization in "autistic" magnets, *Phys. Rev. D* **9**(1974) 16261632.